A neuro-fuzzy inference engine for Farsi numeral characters recognition

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A B S T R A C T

Character recognition of Farsi and Arabic texts as an open and demanding problem needs to encounter sophisticated specifications of the characters such as their shapes, continuity, dots and also, different fonts. Utilizing fuzzy set theory as a tolerant approach toward uncertainty and vagueness and artificial neural networks as a machine learning method in this paper, we propose a neuro-fuzzy inference engine to recognize the Farsi numeral characters. This engine takes holistic approach of character recognition through the comparison of the unknown character’s features with the features of the existing characters that itself is characterized through Mamdani inference engine on fuzzy rules which is largely enhanced with a multi layer perceptron neural network’s learning on features of the different fonts’ characters which leads to more comprehensive recognition of Farsi numeral characters in the proposed system. Having applied this novel engine on a dataset of unknown numeral characters consisted of 33 different Farsi fonts, it yielded more accurate results than the corresponding researches. The recognition rates of unknown numeral characters are greater than 97% except for Farsi character 4, so as the proposed schema could not score a better result than 95% for this numeral character which implies its recognition is still in need of more enhancement.

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1. Introduction

Recent developments in electronic publishing has brought up the need to transformation of paper-based and printed documents to digital formats, hence, text recognition has been highlighted further (Broumandnia & Shanbehzadeh, 2007). Also, extensive applications of character recognition in recognizing the numbers in bank checks (Morita et al., 2000), the car plate numbers (Pan, Ye, & Zhang, 2005), etc. have caused varieties of new systems such as optical character recognition (OCR) system to develop (Al-Alawi, 2004). In character recognition literature, several different methods such as decision tree (Amin, 2000; Nagabhushan & Pai, 1999), fuzzy set theory (Cheok, Jian, & Chng, 2008; Chi, Suters, & Yan, 1996; Hanmandlua, Mohanb, Chakrabortyc, & Goyald, 2003; Pati & Sontakke, 2007), artificial neural networks (Goltsev & Rakhkovskij, 2005; Lauer, Suen, & Bloch, 2007), support vector machines (Camastra, 2007; Malon, Uchida, & Suzuki, 2008), hidden Markov model (Amor & Amara, 2005; Bunke, Roth, & Schukat-Talamazzini, 1995; Khorsheed, 2007; Theeramunkong & Wongtapan, 2005) or any other significant hybridization of these methods (Nagabhushan & Pai, 1999; Ping & Lihui, 2002) are exploited.

Document image analysis has been accentuated as one of the main research issues for about three decades and several methods for recognition of Indian (Al-Omari & Al-Jarrah, 2004; Pal & Chaudhuri, 2004), Thai (Theeramunkong & Wongtapan, 2005), Chinese (Hung, Luk, Yeung, Chung, & Shu, 2005; Li, Tan, Ding, & Liu, 2004; Su, Zhang, Guan, & Huang, 2009; Zhao, Chi, Shi, & Yan, 2003), and Japanese (Itoh, 1995; Kimura, Wakabayashi, Tsuruoka, & Miyake, 1997; Srikanta, Hong, & Srikanta, 1997) documents have been proposed. Also, in spite of numerous researches on recognition of Farsi and Arabic texts (Al-Alawi, 2004; Al-Muhtaseb, Mahmoud, & Qahwaji, 2008; Azmi & Kabir, 2001; Broumandnia & Shanbehzadeh, 2007; Dehghan, Faez, Ahmadi, & Shridhar, 2001; Mozaffari, Faez, Märgner, & El-Abed, 2008; Solimanpour, Sadri, & Suen, 2006; Ziaratban, Faez, & Faradji, 2007), the researchers’ inability in reaching a consensus has made these researches to be continued, so as it is still an open and demanding problem, resulting in new methods of Farsi and Arabic texts recognition. Considering the fact that more than 30% of the world populations talking in the various types of Farsi, Arabic and Urdu dialects use Farsi alphabets and numbers in their written texts, so analyzing and recognizing the Farsi texts are highly crucial (Baghshah, Shoukari, & Kasaei, 2005). On the other hand, the specifications of characters in respect of their shapes, continuity, dots and different fonts in these languages make their recognition more complicated in comparison to other ones in the world (Broumandnia & Shanbehzadeh, 2007).
The character recognition researches can be classified to two various types as: analytical approach and holistic approach (Mitchell & Yan, 2004). The former comprises the pattern verification, momentum measurement and mathematical conversions. In this approach, the pattern is presented as a multi-dimensional vector in a so-called “pattern space” and the purpose will be segmenting the pattern space to some regions called “class”. On the other hand, in the holistic approach, the character is considered as a unique and invisible entity. In this approach, the aim is recognizing the character using its features. This approach comprises two phases; in the first phase, the character features are identified and during the second one, the character is recognized by comparing its features with the features of the existing characters (Ho & Leedham, 2001).

The analytical approach itself is consisted of two other approaches: statistical and deterministic. In statistical approach, the probability density functions are estimated for the patterns distribution in each class, using parametric and non-parametric methods, and consequently the unrecognized samples are classified using such criteria as the minimum error and the minimum risk. On the other hand, in deterministic approach, no specific statistical feature is considered for the patterns and instead, the distance and discriminant functions are usually utilized. Some classification methods such as minimum distance, template matching and K-nearest neighborhood could be mentioned in this regard. Overall, the analytical approach mostly depends on the use of statistical methods such as Markov models, principle component analysis and hierarchical cluster analysis (Bunke et al., 1995; Fang & Alouani, 2002).

Not considering the alphabets and numbers structure in their identification is the prime weakness of the statistical methods. Conversely, the holistic approach, comprising structure and syntax analysis, identifies the new patterns by the help of the previous structures of the alphabets and numbers as well as the features of these structures (Fang & Alouani, 2002). In this approach, each pattern is considered as a set of sub-patterns or principal elements and their interrelations (Dimauro, Impedovo, Pirlo, & Salzo, 1997). Pattern definition in this approach is similar to sentence definition in a language grammar. Namely, the principal elements are like alphabets and a set of syntactic rules helps to define each pattern of a class through various combinations of those elements. According to the geometrical characteristic of the pattern is one of the essential features to be considered in this approach and artificial neural networks (ANNs) are sensible instruments for this purpose. Comparing the strengths and weaknesses of two discussed approaches, analytical and holistic, we choose the holistic approach in this paper. As discussed before, this approach determines the identity of the characters by comparing the features of their new samples with those of the old ones.

The main objective of this paper is to recognize the printed Farsi numeral characters, considering 33 different Farsi fonts through proposing a neuro-fuzzy inference engine. Considering a distinct class for each number and having particular structures in shape, some numbers incorporate uncertainty in their assignment to a specific class. Regarding this fact, we exploit fuzzy set theory to quantify the significant existing uncertainty. The proposed engine first extracts the numbers’ features through the holistic approach which compares the unknown character’s features with the features of the existing characters that itself is characterized through Mamdani inference engine on fuzzy rules which is largely enhanced with a multi layer perceptron neural network’s learning on features of the different fonts’ characters which lead to more comprehensive recognition of Farsi numeral characters in the proposed system. In fact, we have utilized a neuro-fuzzy system to recognize the printed Farsi numeral characters, considering 33 different Farsi fonts.

This paper is organized as follows. In Section 2, the structure analysis methodology is discussed. Then, we propose a neuro-fuzzy system and its usage results on 33 different Farsi fonts are analyzed. At last, the whole discussion is concluded, comparing the new results with the previous studies and analyzing the improvements for Farsi numeral characters recognition.

2. Structural analysis methodology

Doing the preprocessing work, each number goes through a three stepped process; noise elimination, thinning and feature extraction, successively (Broumandnia & Shanbehzadeh, 2007; Mitchell & Yan, 2004). Noise in number recognition is a small blob (in the size of a pixel or a dot) which is resulted from various handwriting styles or small dirt around the hand-written number. Utilizing printed numbers as the inputs to our system in this paper, we sensibly assume that our numbers are free from noise and so, two steps from preprocessing work can be omitted. Thinning is a necessary and important step for a correct stroke extraction. After thinning, feature points are extracted (Mitchell & Yan, 2004).

In general, the feature points comprise three types of points as: terminal points (T), branch points (B) and cross-points (C). Utilizing these feature points, each pattern can be segmented to different regions. In number recognition process, the numbers can be segmented to several regions, considering their feature points. Then, each region can be described by a fuzzy set, considering their fuzzy characteristics. This method can simplify and accurately recognize the recognition process. Considering numeral characters as the main target to identify, we focus on their recognition from now on.

Numbers’ geometrical characteristics are essential factors helping in their recognition and classification. These characteristics can be fused with feature extraction techniques, making the numbers classification based on their shapes possible. For any feature extraction process, the data should be pre-processed beforehand. For gathering the generic information and features about the data, their geometric characteristics and criteria should be extracted. The geometric characteristics include such criteria as pixel density, line, and point. We have utilized numbers’ structure analysis method for recognizing the numbers. As discussed earlier, in numbers’ structure analysis, we need the information about three regions of a number, namely: terminal, branch and cross-points (Fang & Alouani, 2002). Also, any number is made of three essential components: straight line, loop and orientation. Considering this approach as well as segment approach in structure analysis, we can define a fuzzy status for each number including orientation, curvature and number’s weight parameters. We throw more light on these parameters in following.

2.1. Orientation

Orientation is a parameter of arcs and lines, as described below:

1. Arcs' orientation

Arc orientation parameter is indicated using an angle between the principal and vertical axes and is a number between 0° and 360°. By arc in this paper, we mean the regions of Farsi numbers of 2, 3, 4, 5 and 6 which are written using some arcs. Considering all possible orientation statuses in numbers, we define four fuzzy numbers labeled as up, down, right and left, indicated in Fig. 1 by U, D, R and L, respectively. Determining the arc orientation parameter, the discussed angle, we connect the two end points of the arcs using a straight line and the resulting angle between the line and the vertical axis is labeled θ. Finally, we measure the resulting angle between the perpendicular line on the connecting line and the horizon, as the intended parameter.
As presented in Fig. 1, if the orientation angle is between $0^\circ$ and $15^\circ$, the orientation state belongs to "Right" fuzzy set with membership degree of 1. On the other hand, if the aforementioned angle is between $165^\circ$ and $195^\circ$, the orientation state belongs to "Left" fuzzy set with membership degree of 1. In order to cover the marginal angles, like $45^\circ$, which can be assigned to, for example both right and up fuzzy sets simultaneously, we utilize both fuzzy sets' parameters to describe them. Concerning the fact that the precise measurement of the orientation is not possible, using fuzzy concept, we can reach sensible approximation in our recognition work.

On this basis, we can present an arc fuzzy set as

$$
\mu_{Arc} = \mu(h) = (1, 0, 0, 0, 0)
$$

As an example, discussed above, for an orientation angle between $0^\circ$ and $15^\circ$, we will have:

$$
\mu_{Arc} = (1, 0, 0, 0, 0)
$$

Determining the lines' statuses, we utilize four fuzzy sets of: horizontal, negative slope, vertical and positive slope (Montazer & Jafari, 2006). We have lines in all Farsi numbers except Farsi number 5. These lines are presented using the four aforementioned fuzzy sets, as depicted in Fig. 2.

So, the fuzzy set presenting a line can be shown as

$$
\mu_{Line} = \mu(c) = (1, 0, 0, 0, 0)
$$

As an example, for an angle of $\gamma \approx 27^\circ$, based on Fig. 2, we will have:

$$
\mu_{Line} \approx (0, 0, 0.13, 0.37, 0)
$$

2.2. Curvature

Concerning the fact that absolutely precise recognition of the numbers' figures' states as straight line, arc (semi-loop) and loop is not simply possible, by considering three fuzzy sets, describing the aforementioned uncertain states, we can determine them more precisely.

Reaching the degree of curvature, we can utilize following criterion:

$$
C = D/L
$$
where $C$, $D$ and $L$ represent the curvature degree, the distance between two nodes, and the length of the stem connecting two nodes, respectively. If $C = 0$ then $D \approx 0$ and the membership degree of loop-likeness will be 1. On this basis, we can present the curvature fuzzy set as

$$\mu_{\text{Curvature}} = \mu(C) = (\mu_{\text{Loop}}, \mu_{\text{Semi-Loop}}, \mu_{\text{Line}})$$

(4)

Fig. 4 depicts an overall view of our proposed fuzzy system. As shown in this figure, the fuzzy system recognizes the numbers based on three inputs, namely, curvature, line and arc.

3. Rule extraction using fuzzy sets

Extracting the rules using aforementioned fuzzy sets, firstly, the Farsi numbers of 1–9 are presented as the combinations of line, arc and loop, for which their fuzzy sets have already been defined. As Fig. 5 shows, Farsi numbers of 1–9 have been segmented to different segments, which will be discussed next in this paper.

Based on the discussed fuzzy sets in Figs. 1–3, Table 1 describes how Farsi numbers of 1–9 are structured based on the combination of lines, arcs and loops.

Considering the 1st segment, we can conclude that Farsi numbers 1, 2, 3, 4, 7 and 8 could be categorized in one class, however as the line angle in Farsi number 8 is different with the corresponding angle in other numbers, this number would be assigned to a different class. In respect of 2nd segment, we can see that among the numbers with a line as their 1st segments, only Farsi number 7 has line in its 2nd segment. So, Farsi number 7 also would be assigned to a different class. Among the Farsi numbers 1–4, since number one has only one segment and Farsi number 3 is the only number including three segments, so these numbers would be assigned to two different classes as well. Contrasting Farsi numbers 2 and 4 in respect of their arc orientations, we can see that arc orientation of Farsi number 2 is Up, however this is Right for Farsi number 4. On this basis, Farsi numbers 2 and 4 are also assigned to two different classes as well. Farsi numbers 5, 6 and 9 comprise some exclusive features, paving the way for assigning them to three different classes.

In conclusion, the nine Farsi numbers are assigned to nine distinct classes. On this premise, for recognizing the numbers in various fonts, we need to locate the corresponding class of the number. Increasing the accuracy of recognition results, some thinning works are recommended and some small variations are made on numbers. Fig. 6 shows some examples of thinning work on Farsi numbers 4 and 6.

In Farsi, numbers 2, 4, 5 and 6 have various shapes, using different fonts. Making our system comprehensive enough in respect of fonts, we need to also take into account these different shapes as well (Fig. 7).

The accurate combinations of Farsi numbers 1–9, in respect of their structures, are presented in Table 2. Covering all possible structural states of the Farsi numbers, we reach 14 different states,

![Fig. 4. General view of the proposed fuzzy system.](image)

![Fig. 5. The combinations of Farsi numbers 1–9 in respect of line, arc and loop.](image)

![Fig. 6. Thinning works on numbers.](image)
namely, two states for Farsi number 2, three states for each of the Farsi numbers 4 and 6 and one state for each of the remaining numbers.

Tables 3–6 present examples of structural combinations in the Farsi numbers. In these tables, rows cover the 1st segments while columns cover the 2nd segments.

Concerning the following characteristics, the fuzzy rules are extracted:

If \( X_{1p} \) is \( A_i \) and \( X_{2p} \) is \( B_j \) and \( X_{3p} \) is \( C_k \) then
\[
X_p \text{ belongs to } W_{ijk} \text{ with } CF = CF_{ijk} \tag{5}
\]
where \( X_p \) is an input pattern in which \( X_{1p} \) is the 1st segment, \( X_{2p} \) is the 2nd segment and \( X_{3p} \) represents the 3rd segment. \( A_i \) in which \( i = 1, 2, 3 \), represents that its 1st segment is line, arc or loop. Similarly, \( B_j \) (\( j = 1, 2, 3 \)) and \( C_k \) (\( k = 1, 2, 3 \)) represent the same states for 2nd and 3rd segments as well. For example for Farsi number 2, the rules are:

\[
\begin{align*}
\text{If } X_{1p} \text{ is Line (N) and } X_{2p} \text{ is Arc (Up) and } X_{3p} \text{ is null then } X_p \text{ belongs to } W_{ijk} = 2. \\
\text{If } X_{1p} \text{ is Line (V) and } X_{2p} \text{ is Arc (Up) and } X_{3p} \text{ is null then } X_p \text{ belongs to } W_{ijk} = 2. \\
\text{If } X_{1p} \text{ is Line (P) and } X_{2p} \text{ is Arc (Up) and } X_{3p} \text{ is null then } X_p \text{ belongs to } W_{ijk} = 2. \\
\text{If } X_{1p} \text{ is Line (N) and } X_{2p} \text{ is Arc (Up and Right) and } X_{3p} \text{ is null then } X_p \text{ belongs to } W_{ijk} = 2. \\
\text{If } X_{1p} \text{ is Line (V) and } X_{2p} \text{ is Arc (Up and Right) and } X_{3p} \text{ is null then } X_p \text{ belongs to } W_{ijk} = 2. \\
\text{If } X_{1p} \text{ is Line (P) and } X_{2p} \text{ is Arc (Up and Right) and } X_{3p} \text{ is null then } X_p \text{ belongs to } W_{ijk} = 2. \\
\text{If } X_{1p} \text{ is Line (N) and } X_{2p} \text{ is Arc (Up and Left) and } X_{3p} \text{ is null then } X_p \text{ belongs to } W_{ijk} = 2. \\
\text{If } X_{1p} \text{ is Line (V) and } X_{2p} \text{ is Arc (Up and Left) and } X_{3p} \text{ is null then } X_p \text{ belongs to } W_{ijk} = 2. \\
\text{If } X_{1p} \text{ is Line (P) and } X_{2p} \text{ is Arc (Up and Left) and } X_{3p} \text{ is null then } X_p \text{ belongs to } W_{ijk} = 2. \\
\end{align*}
\]

It can be seen that the 3rd segment in Farsi number 2 is “null”, so the membership degree for this segment can be considered 1. Considering the fact that an accurate recognition of N, V and P in various patterns is not possible, each pattern can be assigned to fuzzy sets of N, V and P under a membership degree in the range of 0 and 1. In this regard, we can consider some membership functions for three aforementioned segments as: \( \mu(X_{1p}) \), \( \mu(X_{2p}) \) and \( \mu(X_{3p}) \). If the 3rd segment value is null, we set its membership degree as 1: \( \mu(X_{3p} = \text{null}) = 1 \). On this basis, we extract 128 fuzzy rules for Farsi numbers 1–9. Table 7 presents the association of these rules to the numbers.

Based on all we have presented in this paper hitherto, we not only determine that with how much membership degree a segment of a Farsi number is a vertical line or line with a negative slope, etc., but also we discuss its shape in respect of a line, arc or loop by some other membership degrees that are assigned to

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**Table 2**
Accurate structural combinations of Farsi numbers.

<table>
<thead>
<tr>
<th>State counter</th>
<th>Farsi numbers (in various fonts)</th>
<th>Numbers in English</th>
<th>1st Segment</th>
<th>2nd Segment</th>
<th>3rd Segment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>Line</td>
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<td>–</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>Line</td>
<td>Arc</td>
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<td>–</td>
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<tr>
<td>3</td>
<td>2</td>
<td>Line</td>
<td>Line</td>
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<td>–</td>
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<tr>
<td>4</td>
<td>3</td>
<td>Line</td>
<td>Arc</td>
<td>Arc</td>
<td>–</td>
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<tr>
<td>5</td>
<td>4</td>
<td>Line</td>
<td>Arc</td>
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</tr>
<tr>
<td>6</td>
<td>4</td>
<td>Line</td>
<td>Line</td>
<td>Line</td>
<td>Arc</td>
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<td>7</td>
<td>4</td>
<td>Arc</td>
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<td>8</td>
<td>5</td>
<td>Arc</td>
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<td>9</td>
<td>6</td>
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<td>10</td>
<td>6</td>
<td>Line</td>
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<td>11</td>
<td>7</td>
<td>Arc</td>
<td>Line</td>
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<tr>
<td>12</td>
<td>7</td>
<td>Line</td>
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<td>13</td>
<td>8</td>
<td>Line</td>
<td>Line</td>
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<tr>
<td>14</td>
<td>9</td>
<td>Loop</td>
<td>Line</td>
<td>–</td>
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</tbody>
</table>

**Table 3**
Arc as the 1st segment and line as the 2nd segment.

<table>
<thead>
<tr>
<th>Arc (R)</th>
<th>Arc (U)</th>
<th>Arc (L)</th>
<th>Arc (D)</th>
<th>Arc (R)</th>
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</thead>
<tbody>
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**Table 4**
Line as the 1st segment and Arc as the 2nd segment.

<table>
<thead>
<tr>
<th>Arc (R)</th>
<th>Arc (U)</th>
<th>Arc (L)</th>
<th>Arc (D)</th>
<th>Arc (R)</th>
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**Table 5**
Line as the 1st and 2nd segments.

<table>
<thead>
<tr>
<th>Line (H)</th>
<th>Line (N)</th>
<th>Line (V)</th>
<th>Line (P)</th>
<th>Line (H)</th>
</tr>
</thead>
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**Table 6**
Arc as the 1st and 2nd segments.

<table>
<thead>
<tr>
<th>Arc (R)</th>
<th>Arc (U)</th>
<th>Arc (L)</th>
<th>Arc (D)</th>
<th>Arc (R)</th>
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Fig. 7. Various shapes of numbers 4 and 6 in Farsi.
it. On this basis, we utilize two distinct membership degrees to determine the Farsi number status, firstly how much it is a line, arc or a loop and afterwards, how much it is a vertical line, a line with negative or positive slope or a horizontal line. We call the former “primary membership degree” and the latter is called “secondary membership degree”. The former membership degree is settled using $C = D/L$ as in Fig. 3. Calculating the primary membership degree for Farsi number 2 is done, using Mamdani multiplication as follows:

If $X_{1p}$ is Line (N) and $X_{3p}$ is Arc (Up) and $X_{3p}$ is null, then

$$
\mu(C) \cdot \mu(\gamma) \cdot (\mu(0) \cdot (\mu(X_{3p} = \text{null}) = 1)
$$

(6)

Determining secondary membership degree, we utilize Mamdani implication (Mamdani, 1976) as follows:

$$
\mu_{Q_{up}}(X_{1p}, X_{2p}, X_{3p}, W) = \mu(X_{1p}) \cdot \mu(X_{2p}) \cdot \mu(X_{3p}) \cdot \mu(W)
$$

(7)

In addition, we have selected Mamdani multiplication inference engine, out of Mamdani minimum, Lukasewich, Zadeh and Dennis-Rosher inference engines, so as, we have:

$$
\mu_{Q_{up}}(Y) = \frac{\max}{i=1} \left\{ \sup_{X \in U} \left[ \mu_{X}(X) \cdot \prod_{j=1}^{n} \mu_{X_j}(X_j) \cdot \mu_{Y}(Y) \right] \right\}
$$

(8)

Causes of selecting this inference engine are as follows:

- The associated rules for numbers are local rather than global. In other words, if the 2nd segment of a number includes an Arc (Up), an Arc (Up-Right) or an Arc (Up-Left), then we can conclude that the Farsi number is 2 and we cannot reach any other conclusions upon another status.
- Considering the fact that every rule plays a significant role in determining the conclusion, we utilize inference based on separate rules and union combination.
- Utilizing Mamdani inference engine can simplify the calculations and speed up the calculation process.

Also, we have utilized minimal operators for t-norm, making our approach more conservative and maximal operators for s-norm.

### 4. Farsi numeral characters data set

Implementing our proposed system, we set up a data set including the Farsi numbers 1–9, using various Farsi fonts. Table 8 presents the Farsi numbers and their fonts in the data set. As presented in Table 8, we have utilized 33 various fonts for presenting Farsi numbers. Fig. 8 depicts examples of these fonts for the Farsi numbers 4 and 6.

### 5. A neuro-fuzzy system for Farsi numeral characters classification

After proposing a fuzzy inference engine for Farsi characters classification, we enhance the recognition capability of the system with a neural network to classify the characters of the different fonts. We used a multi layer perceptron (MLP) neural network to equip the proposed engine with learning the features of the different font’ characters, so as, the system recognition capability would be enhanced in this way.

The input pattern of this network consists of a character’s feature vector which includes the membership values of the features such as line, arc and loop. All the input pattern’s features have been passed to nine hidden nodes because of the nine Farsi digits from 1

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Extracted rules for Farsi numbers and their associations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of associated rules</td>
<td>Number of Farsi</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>27</td>
<td>3</td>
</tr>
<tr>
<td>45</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>21</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 8</th>
<th>Farsi numbers data set using various fonts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farsi numbers</td>
<td>Font</td>
</tr>
<tr>
<td>١</td>
<td>Traditional Arabic</td>
</tr>
<tr>
<td>٢</td>
<td>Lotus</td>
</tr>
<tr>
<td>٣</td>
<td>B Esfahan</td>
</tr>
<tr>
<td>٤</td>
<td>Titre</td>
</tr>
<tr>
<td>٥</td>
<td>Simplified Arabic</td>
</tr>
<tr>
<td>٦</td>
<td>Zar</td>
</tr>
<tr>
<td>٧</td>
<td>Shiraz</td>
</tr>
<tr>
<td>٨</td>
<td>Compset</td>
</tr>
<tr>
<td>٩</td>
<td>Mitra</td>
</tr>
<tr>
<td>١</td>
<td>Arial</td>
</tr>
<tr>
<td>٢</td>
<td>Sina</td>
</tr>
</tbody>
</table>

Fig. 8. Utilizing various fonts for Farsi numbers 4 and 6.
to 9, so as each node computes the character’s compatibility with a digit. After calculation of all the hidden nodes are completed, their results are compared and the most similar one will be identified through selecting the digit with maximal compatibility. Fig. 9 depicts an overall view of the proposed neuro-fuzzy system.

Each node in the first layer calculates the degree of association of input pattern to corresponding class using the membership function $\mu_{i,t}(x)$, where $i$ represents the $i$th pattern and $t$ represents the $t$th class. Nodes in the second layer of the neuro-fuzzy system calculate the multiplication of the membership degrees, resulted from the first layer. Considering the utilization of Mamdani multiplication engine, we use multiplication operator for this purpose

$$\mu_{i,t}(x) = \prod_{t=1}^{m} \mu_{i,t}(x)$$

(9)

where $i$, $t$ and $m$ represent the $i$th pattern, the $t$th class and total number of classes, respectively. The output node in $i$th network determines the membership degree of $x$ to $i$th class:

$$\mu_{i}(x) = \max_{j=1}^{9}(\mu_{i,j}(x))$$

(10)

To test the proposed engine, we have utilized some random data selected from the aforementioned data set (Fig. 10).

Fig. 10 depicts some of the randomly selected numbers from the data set. The system starts recognizing the numbers from upper-left side of the character image going towards right-down side of it. Afterwards, thinning work will be done on resulted characters. Identifying the cross-points in the numbers and considering the aforementioned fuzzy sets in previous sections, the membership degrees of each number to each of the classes are calculated and finally, the corresponding class for each number will be resulted. We have utilized the following formula in calculating the recognition rate for various Farsi numbers to examine the proposed method performance in recognizing the numbers:

$$\text{Recognition rate} = \frac{\text{Number of correctly classified samples}}{\text{Total number of samples}} \times 100$$

(11)

After calculating the recognition rates of the proposed system for the Farsi numeral characters, as presented in Fig. 11, the accuracy level of recognition was not satisfactory enough, stimulating us to utilize a new learning approach which is described below.

Considering the fact that in this problem there is no specific desired output to be compared with the real system output and finding the degree of error (the approach which is used in supervised artificial neural networks), we have applied reinforced learning as our possible choice for learning approach

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Fig. 9. Patterns classification using fuzzy system.

Fig. 10. Randomly selected data for testing the system.
the approach which is mainly used in unsupervised artificial neural networks).

Al-Alawi has utilized a reinforced learning approach in classification (Al-Alawi, 2004). In this problem, we have utilized the reinforced learning approach with some small modifications for our case. In this learning algorithm, which is aimed to make similar vectors for each class, the system is reinforced up on a desirable classification, however it is weakened up on an undesirable classification. Having trained the vectors for each class, we will utilize them in creating a fuzzy rule set for classifying the new patterns into right classes.

In order to train the system, for each class, we have applied the corresponding numeral character of the class in various fonts. On this basis, we have used nine patterns in training. Fig. 12 shows some examples of four of these patterns.

During the training process, the vectors of the classes get corrected gradually, resulting in a correct vector for each class at last. By vector, we mean the membership degrees of the elements of each number in a class. For example, the resulted membership degrees for the Farsi numbers of 2 and 7 after the training process are presented in Table 9.

Based on the resulted vectors for each class, the fuzzy rules are extracted after the training process. As an example, for number 7, taking into account some tolerances for its membership degrees, we reached the following rules:

<table>
<thead>
<tr>
<th>Number in Farsi</th>
<th>First element</th>
<th>Second element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line (N) = 0.73</td>
<td>Arc (L–U) = 0.68</td>
<td></td>
</tr>
<tr>
<td>Line (V) = 0.86</td>
<td>Arc (U) = 0.89</td>
<td></td>
</tr>
<tr>
<td>Line (P) = 0.21</td>
<td>Arc (L–U) = 0.35</td>
<td></td>
</tr>
<tr>
<td>Line (N) = 0.84</td>
<td>Line (N) = 0.12</td>
<td></td>
</tr>
<tr>
<td>Line (V) = 0.36</td>
<td>Line (V) = 0.53</td>
<td></td>
</tr>
<tr>
<td>Line (P) = 0.03</td>
<td>Line (P) = 0.91</td>
<td></td>
</tr>
</tbody>
</table>
If \( 0.81 \leq \mu_{x_1} \leq 0.87 \) and \( 0.88 \leq \mu_{x_2} \leq 0.94 \) and \( X_{3p} \) is null then \( X_p \) belongs to \( W_{ijk} = 7 \)

\[ \vdots \]

If \( 0 \leq \mu_{x_{1p}} \leq 0.06 \) and \( 0.09 \leq \mu_{x_{2p}} \leq 0.15 \) and \( X_{3p} \) is null then \( X_p \) belongs to \( W_{ijk} = 7 \)

First rule presents the state when the first segment of the Farsi number 7 is a line with negative slope and its second segment is a line with positive slope and the last rule presents the state when the first segment is a line with positive slope and the second segment is a line with negative slope. It is note worthy that there are nine possible states for Farsi number 7. Extracting the fuzzy rules for each class, we tested the system again utilizing new input patterns. The resulting recognition rate is as follows in Fig. 13.

Having trained the neuro-fuzzy system and improved the fuzzy rules, the recognition rate enhanced a large extent, as presented in Fig. 13. However the system shows some ineffectiveness in recognizing the Farsi number 4. Dealing with this problem, we changed the sequence of input data to the neuro-fuzzy system and trained the system using new order of input data. Fig. 14 depicts the consequence of this change in the recognition rates of the characters.

The steps of training process are as follows:

Step 1: Considering \( i = 1 \) and \( j = 1 \), all patterns of the \( j \)th class starting from \( i \)th one are utilized for training.

Step 2: Reaching the last pattern of the \( j \)th class, then \( j = j + 1 \) and the step 1 would be repeated for the patterns of this class.

Step 3: Aforementioned steps are repeated for all classes and would be ended by reaching the last pattern of the last class.

Step 4: Having finished the training phase, fuzzy rules are extracted for each class based on their determined characteristics.

Step 5: Considering the other classes' rules, the rules for each class are summarized. This process continues till the final class rules extraction.

Steps 1–3 are belonged to training phase of the process and steps 4 and 5 are related to reinforced learning phase of the process. Reaching better results, we segmented the input data to two categories, one for training phase (TN) and the other for reinforced learning (TR), respectively. Fig. 15 presents the process of this segmentation.

Number of training data for each number in our problem is maximum 33. Making some changes in the number of training data, we have calculated the recognition rates for each of the aforementioned classes. Fig. 16 presents the recognition rate for class 1 which is belonged to Farsi number 1.
As indicated in Fig. 16, when the number of input data for training is 12 and the number of input data for reinforced learning is 9, the recognition rate of Farsi number 1 reaches the maximum level of 1 most quickly. Table 10 presents the recognition rates of Farsi number 1 for various combinations of input data.

The presented proportions in Table 10 are related to recognition rates of more than 0.99. On this basis, we can see that the learning rate for Farsi numbers of 1 and 5 are the highest, while this rate for the Farsi number 4 is the least.

As the Table 11 implies the comparison of the proposed neuro-fuzzy engine results with corresponding researches, it could be concluded that hybridizing Mamdani inference on fuzzy rules with multi layer perceptron neural network learning on the characters features in different fonts. Experimental results on a dataset consisted of 33 different Farsi fonts, show that this novel engine could score better results against the corresponding researches.

The proposed engine provided recognition rates between [97% and 100%] interval for the Farsi numeral characters except Farsi numeral character 4, so as this character’s 95% recognition rate still needs more enhancement. In fact, the recognition capability of the proposed neuro-fuzzy engine which is mostly based on the Mamdani inference on fuzzy rules, has been largely augmented through learning on the characters features via a multi layer perceptron neural network, so as the results of the proposed engine analysis versus the corresponding researches show that the hybridizing neuro-fuzzy system yields better performance in Farsi numeral characters recognition.

Table 10
The proportion of number of training and the reinforced learning data.

<table>
<thead>
<tr>
<th>Number (Farsi)</th>
<th>Number of data used for training</th>
<th>Number of data utilized in reinforced learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 11
Comparison between the proposed engine against the corresponding researches.

<table>
<thead>
<tr>
<th>No.</th>
<th>Reference</th>
<th>Classification method</th>
<th>Recognition rate (% on average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ziaratban, Faez, and Faradji (2007)</td>
<td>Multi layer perceptron</td>
<td>97.65</td>
</tr>
<tr>
<td>2</td>
<td>Solimanpour, Sadri, and Suen (2006)</td>
<td>Support vector machines</td>
<td>97.32</td>
</tr>
<tr>
<td>3</td>
<td>Montazer and Jafari (2006)</td>
<td>Fuzzy set theory</td>
<td>91</td>
</tr>
<tr>
<td>4</td>
<td>The proposed engine</td>
<td>Neuro-fuzzy system</td>
<td>98</td>
</tr>
</tbody>
</table>

Fig. 16. Comparison of recognition rates considering the number of input data for training and reinforced learning.

6. Conclusion

Due to sophisticated specifications of the Farsi and Arabic numeral characters such as their shapes, continuity and also, different fonts, their recognition is still an open and demanding problem. In this paper, a neuro-fuzzy engine was proposed to encounter this problem through holistic approach. This engine exploits Mamdani inference on fuzzy rules which is enhanced with multi layer perceptron neural network learning on the characters features in different fonts. Experimental results on a dataset consisted of 33 different Farsi fonts, show that this novel engine could score better results against the corresponding researches.

The proposed engine provided recognition rates between [97% and 100%] interval for the Farsi numeral characters except Farsi numeral character 4, so as this character’s 95% recognition rate still needs more enhancement. In fact, the recognition capability of the proposed neuro-fuzzy engine which is mostly based on the Mamdani inference on fuzzy rules, has been largely augmented through learning on the characters features via a multi layer perceptron neural network, so as the results of the proposed engine analysis versus the corresponding researches show that the hybridizing neuro-fuzzy system yields better performance in Farsi numeral characters recognition.

References


